LIGO Hanford Schnupp Asymmetry Measurement 20/03/2014 – David Ottaway

By re-analysing the data that was used to measure the PRC length it is possible to gain a measurement of the PRC round trip losses and hence some insight into the Schnupp Asymmetry. (See alog entry 9th March 2014)

By considering the PRC as a simple cavity with a lossy end mirror it is possible to predict the beatnote transfer function results. The effective reflectivity of the cavity is given by:

$$\frac{E_{refl}}{E_{inc}} = r_{prm} - \frac{T_{prm} r_{mich} \exp(i\emptyset_{prc})}{1 - r_{prm} r_{mich} \exp(i\emptyset_{prc})} \tag{1}$$

Where r_{prm} is the reflectivity of the PRM and T_{prm} is the transmission of the PRM, r_{mich} is the effective amplitude reflectivity of the PRM and φ_{prc} is the roundtrip phase of the PRC. This quantity is proportional to the beat-note when this light field is beat against the carrier.

The roundtrip phase of the PRC can be re-written as

$$\emptyset_{prc}(f_{sc}) = \pi + \frac{4\pi f_{sc} l_{prc}}{c} \tag{2}$$

To complete the picture a few more terms need to be added including transfer function of the photodetector, phase shifts associated with any cable delays and the fraction of the electric field that is not mode-matched to the PRC.

$$TF = TF_{PD} \exp(-i2\pi\tau_{cable} f_{sc}) \left(r + \frac{T_{prm} r_{mich} \exp\left(i \frac{4\pi f_{sc} l_{prc}}{c}\right)}{1 + r_{mich} \exp\left(i \frac{4\pi f_{sc} l_{prc}}{c}\right)} + e \right) \tag{3}$$

The time delay τ_{cable} was measured and found to be 142.66 ns. To fit equation 3 using automated complex methods was tricky so I did all the 6 frequencies by hand. An example fit is shown as Figure 1

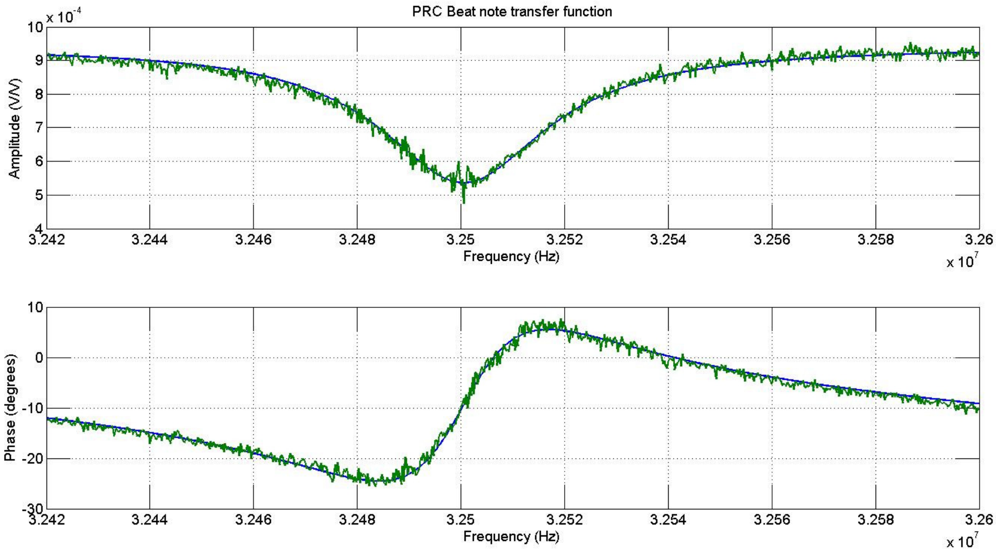


Figure 1 Measured beatnote with hand fit for the resonance centred on 32.5 MHz

The aim of this exercise was to obtain the frequency dependent losses of the cavity and hence the Schnupp Asymmetry. These fits took quite a lot of interpretation so I decided to fit only the phase using the Matlab curve fitting toolbox.

To do this I simplified Equation 3 to the following form:

$$Phase = A + \ angle\left(B - \frac{C\exp\left(i\,\frac{4\pi(f_{sc} - f_{mod} - E)l_{prc}}{c}\right)}{1 - r_{prm}\,C\exp\left(i\,\frac{4\pi(f_{sc} - f_{mod} - E)}{c}\right)}\right) \tag{4}$$

Note the phase delay due to the cable was removed before this fitting was done. The data was fitted to A, B, C and E. The l_{prc} used was the value obtained previously and f_{mod} was the centre resonance frequency.

These important quantities are

$$B = \frac{r_{prm} - e}{T_{prm}}$$

$$C = r_{mich}$$

And E is the offset from the ideal resonance. The fits data are shown below in the Appendix

Three example fits are shown in the following figure

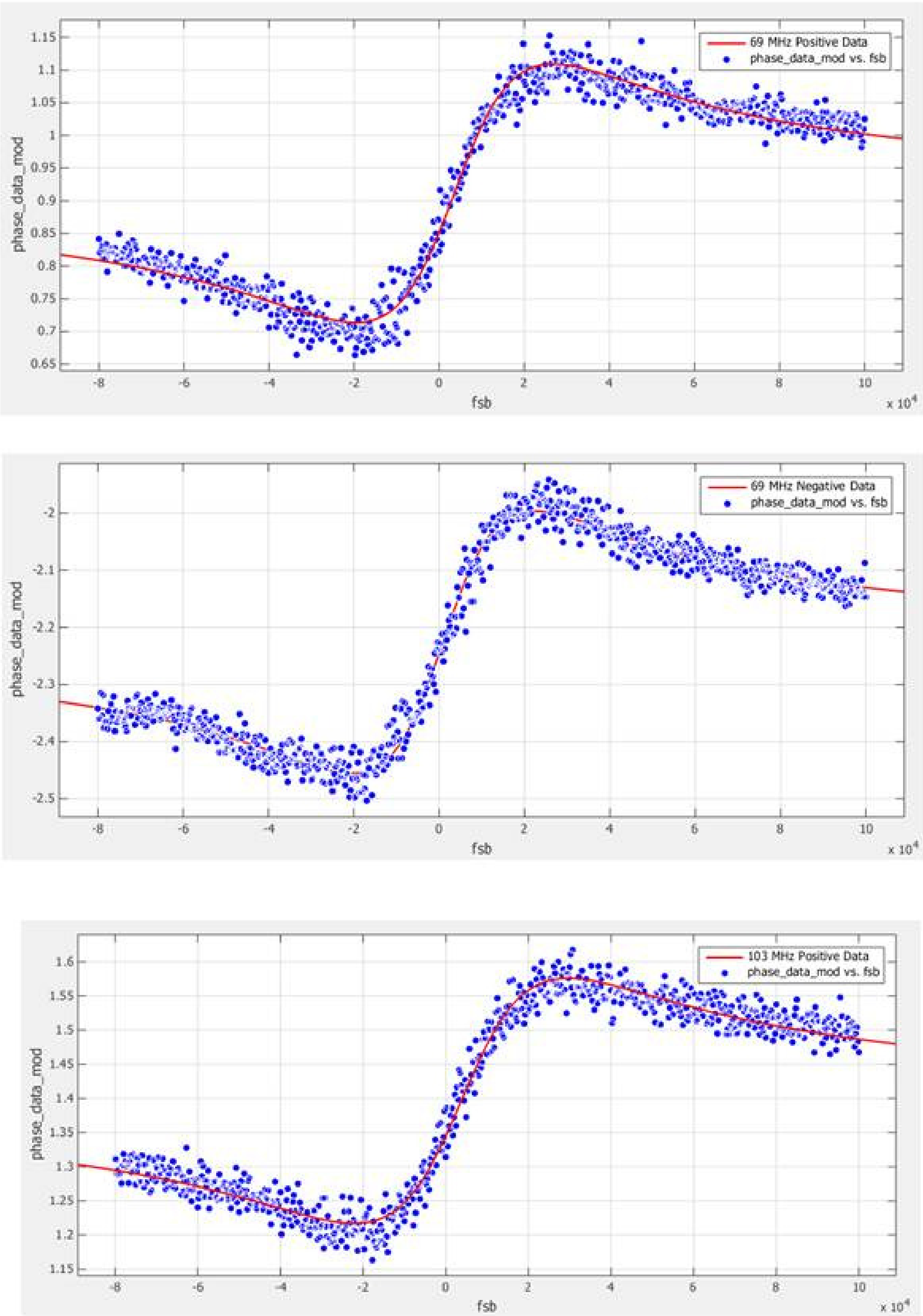
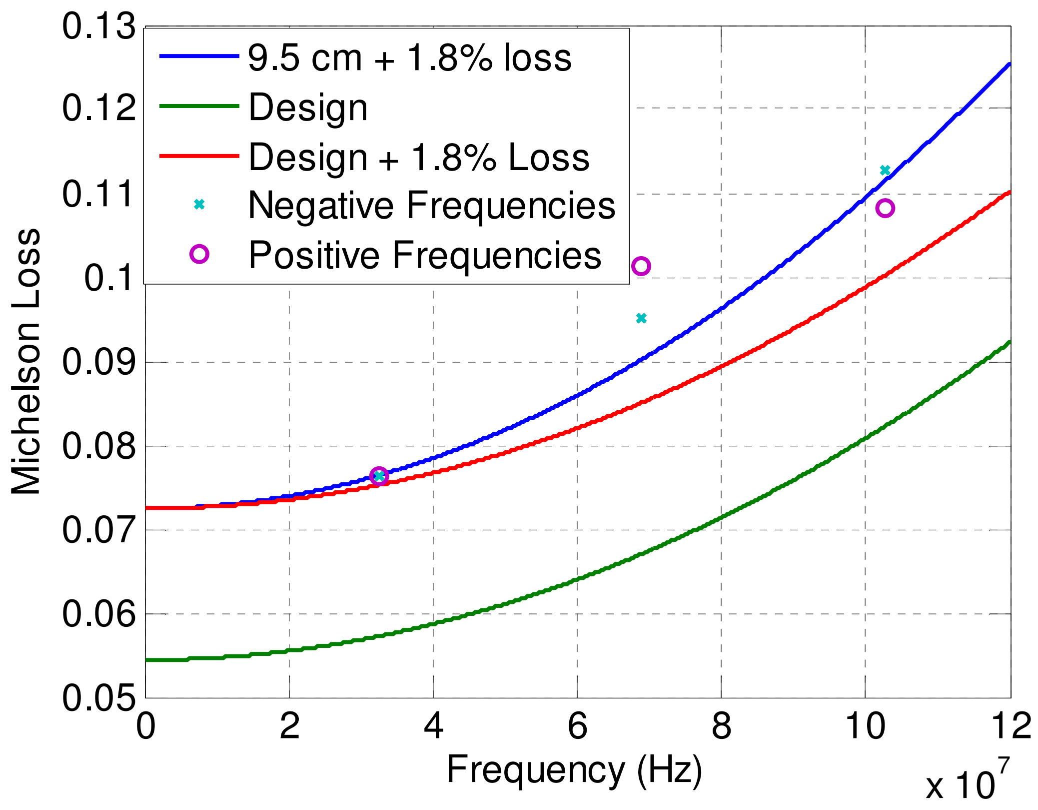


Figure 2 Sample fits -Note the top one does not fit as well. All 5 other fits are similar quality to the botton 2

A table of the measured r_{mich} is shown below vs sub-carrier frequency

Frequency (MHz)	r_{mich}	Error Range
+32.5	0.9611	(0.9605, 0.9617)
-32.5	0.9611	(0.9605, 0.9617)
+68.9	0.948	(0.9467, 0.9493)
-68.9	0.9512	(0.9501, 0.9524)
102.7	0.9443	(0.9426, 0.9461)
-102.7	0.9419	(0.9404, 0.9435)

These values were converted to a Michelson loss not including the R_{prc} and are plotted below against predicted values.



The effective reflectivity of the BS/ITMX/ITMY can be described by the following equation:

$$R_{mich} = (R_{bs})^2 R_y + 2\sqrt{R_x R_y} T_{bs} R_{bs} \cos\left(\frac{4\pi f_{sc} l_{Schupp}}{c}\right) + (T_{bs})^2 R_x \quad (5)$$

At LHO the ITMX is currently an inverted ETMX with no AR coating on the non HR surface. We therefore anticipate a loss of 9.4% from the ITMX and 1.4% from ITMY.

Conclusion: This data hints at the fact that the Schnupp Asymmetry maybe slight to big ~9.5 cm although a repeat of this measurement with the increased signal to noise that is now available to be sure.

Appendix Data Fits

103 MHz Negative Data

General model:

f(x) = A+angle(B-C*exp(-i*2.4165e-006*(x-E))/(1-0.985*C*exp(-i*2.4165e-006*(x-E))))

Coefficients (with 95% confidence bounds):

A = -1.868 (-1.87, -1.866)

B = 40.91 (39.95, 41.87)

C = 0.9443 (0.9426, 0.9461)

E = 1165 (805.5, 1525)

Goodness of fit:

SSE: 0.4942

R-square: 0.9723

Adjusted R-square: 0.9721

RMSE: 0.02625

103 MHz Positive Data

Coefficients (with 95% confidence bounds):

A = 1.396 (1.395, 1.398)

B = 44.23 (43.34, 45.13)

C = 0.9419 (0.9404, 0.9435)

E = 3771 (3449, 4093)

Goodness of fit:

SSE: 0.2819

R-square: 0.9798

Adjusted R-square: 0.9797

RMSE: 0.01983

69 MHz Positive Data

Coefficients (with 95% confidence bounds):

A = 0.9113 (0.9098, 0.9129)

B = 44.6 (43.75, 45.44)

C = 0.948 (0.9467, 0.9493)

E = 3553 (3269, 3836)

Goodness of fit:

SSE: 0.3027

R-square: 0.9811

Adjusted R-square: 0.981

RMSE: 0.02055

69 MHz Negative Data

Coefficients (with 95% confidence bounds):

A = -2.226 (-2.228, -2.224)

B = 41.56 (40.84, 42.29)

C = 0.9512 (0.9501, 0.9524)

E = 891.4 (643, 1140)

Goodness of fit:

SSE: 0.3434

R-square: 0.9831

Adjusted R-square: 0.9831

RMSE: 0.02188

33 MHz Positive Data

Coefficients (with 95% confidence bounds):

A = 3.003 (3.002, 3.004)

B = 42.48 (42.04, 42.91)

C = 0.9611 (0.9605, 0.9617)

E = 1529 (1403, 1655)

Goodness of fit:

SSE: 0.1661

R-square: 0.9936

Adjusted R-square: 0.9936

RMSE: 0.01522

33 MHz Negative Data

Coefficients (with 95% confidence bounds):

A = -0.1693 (-0.1704, -0.1682)

B = 43.07 (42.64, 43.5)

C = 0.9611 (0.9605, 0.9617)

E = 218.4 (93.82, 342.9)

Goodness of fit:

SSE: 0.1561

R-square: 0.9938

Adjusted R-square: 0.9937

RMSE: 0.01475