Temperature Noise

Start from the idea that the temperature of the environment is fluctuating:
1. Assume that the test mass temperature is constant (or at least fluctuating in an uncorrelated way with the distant environment
2. Assume that the fluctuations in the CP/RM are not correlated with the distant environment
3. So the front surface of the test mass sees a fluctuation in temperature of the environment

Relative temperature noise vs relative intensity noise - Stefan-Boltzmann

$$\frac{dP}{P} = 4\,\frac{dT}{T}$$

RIN = 4 RTN

```
In[7]:= Tenv = 293;
```

Radiation Pressure

See equation 2.33 in Stefan Ballmer' s thesis

```
In[20]:= Pinc = sigma epsilon A1 Tenv^4 /.
           {sigma → 5.67 * 10^-8, epsilon → 0.9, A1 → Pi * 0.17^2}
         dzRP = (Pinc / c) / (m (2 * Pi f)^2) RIN /. {RIN → 4 RTN, m → 40, c → 3 * 10^8}
         dzRPIFO = 2 * dz (* 4 optics with uncorrelated noise *)

Out[20]= 34.1463
```

Out[21]= $\frac{2.88312\times 10^{-10}\ \mathrm{RTN}}{f^2}$

Out[22]= $\frac{5.76624\times 10^{-10}\ \mathrm{RTN}}{f^2}$

Expansion of the optic & change in index of refraction - ITMs - Eqn 2.36 in SB Thesis

See equation 2.36, 2.38 in Stefan Ballmer' s thesis

In[66]:=
```
pxy = sigma epsilon Tenv^4 /. {sigma → 5.67 * 10^-8, epsilon → 0.9};
dz = ((1 + eta) alpha (1 - Pi/(2 F) (n - 1)) - Pi/(2 F) dndT) 1/(2 Pi f C1 rho) pxy /. {eta → 0.15,
    alpha → 0.55 * 10^-6, C1 → 2200, rho → 740, dndT → 8.6 * 10^-6, F → 450, n → 1.45}
dzMean = dz (*Nominally do integral in equation 2.39,
but pxy has no spatial dependence, so dzMean = dz *)
dzNoiseITM = dzMean * Sqrt[2] * RIN /.
   RIN → 4 RTN (* Sqrt[2] comes from quadradture sum of 2 ITMs*)
```

Out[67]= $\frac{2.2115\times 10^{-11}}{f}$

Out[68]= $\frac{2.2115\times 10^{-11}}{f}$

Out[69]= $\frac{1.25102\times 10^{-10}\ RTN}{f}$

Expansion of the optic - ETMs - Eqn 2.36 in SB Thesis

See equation 2.36, 2.38 in Stefan Ballmer' s thesis

In[70]:=
```
pxy = sigma epsilon Tenv^4 /. {sigma → 5.67 * 10^-8, epsilon → 0.9};
dz = ((1 + eta) alpha) 1/(2 Pi f C1 rho) pxy /.
   {eta → 0.15, alpha → 0.55 * 10^-6, C1 → 2200, rho → 740}
dzMean = dz (*Nominally do integral in equation 2.39,
but pxy has no spatial dependence, so dzMean = dz *)
dzNoiseETM = dzMean * Sqrt[2] * RIN /. RIN → 4 RTN
```

Out[71]= $\frac{2.32553\times 10^{-11}}{f}$

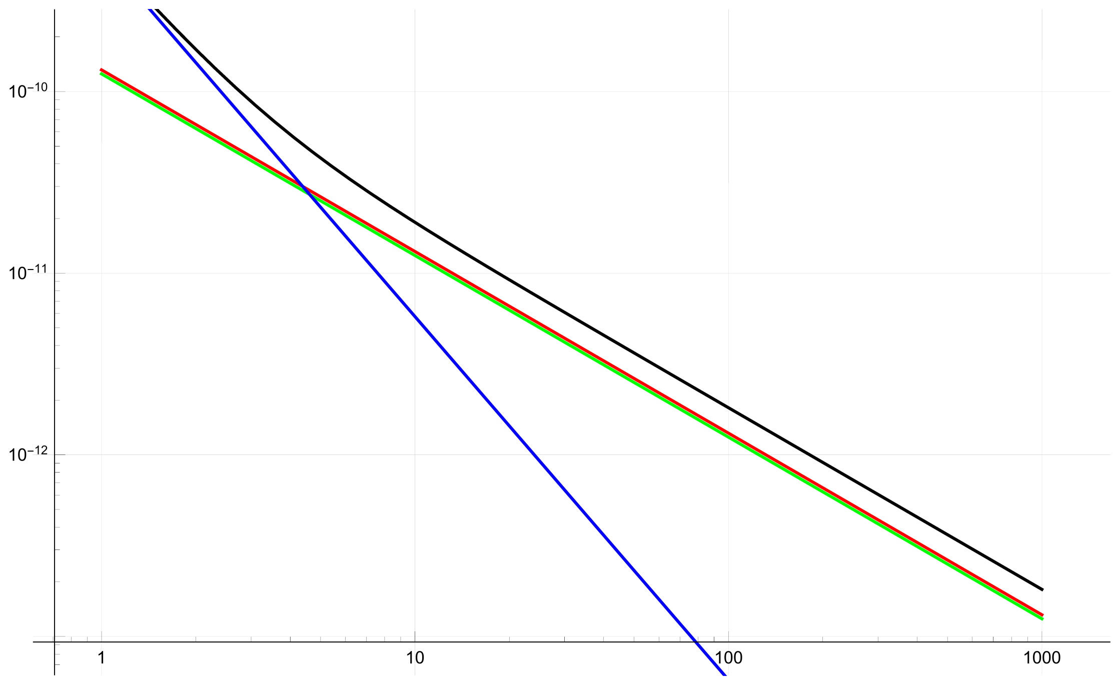
Out[72]= $\frac{2.32553\times 10^{-11}}{f}$

Out[73]= $\frac{1.31552\times 10^{-10}\ RTN}{f}$

In[88]:=
```
TotalTemperatureNoise = Sqrt[dzNoiseITM^2 + dzRPIFO^2 + dzNoiseETM^2];
plot1 = LogLogPlot[dzNoiseETM /. RTN → 1,
   {f, 1, 1000}, GridLines → Automatic, PlotStyle → Red];
plot2 = LogLogPlot[dzNoiseITM /. RTN → 1, {f, 1, 1000},
   GridLines → Automatic, PlotStyle → Green];
plot3 = LogLogPlot[dzRPIFO /. RTN → 1, {f, 1, 1000},
   GridLines → Automatic, PlotStyle → Blue];
plot4 = LogLogPlot[TotalTemperatureNoise /. RTN → 1,
   {f, 1, 1000}, GridLines → Automatic, PlotStyle → Black];

Show[plot1, plot2, plot3, plot4]
```

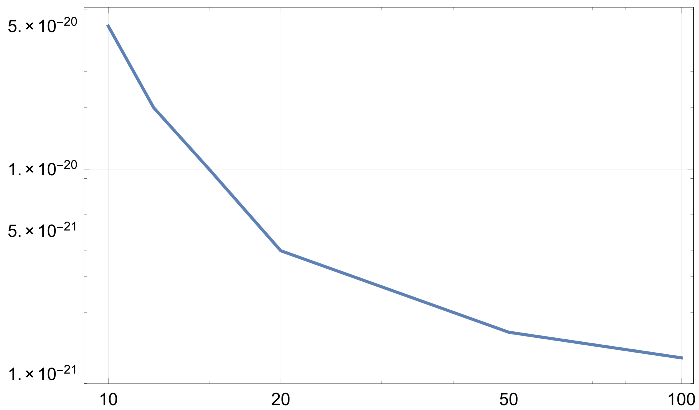
Out[93]=



Maximum Acceptable Displacement Noise

In[197]:=
```
MaxDisplacementNoise = N[{{10, 5 * 10^-20}, {12, 2 * 10^-20}, {15, 10^-20},
    {20, 4 * 10^-21}, {50, 1.6 * 10^-21}, {100, 1.2 * 10^-21}}];
ListLogLogPlot[MaxDisplacementNoise, Joined → True,
 GridLines → Automatic, Frame → True]
```

Out[198]=



Maximum Acceptable Temperature Noise

In[143]:=
```
RTNCoefficient = Sqrt[ Coefficient[dzNoiseITM , RTN] ^2 +
   Coefficient[dzNoiseETM , RTN] ^2 + Coefficient[dzRPIFO , RTN] ^2 ]
```

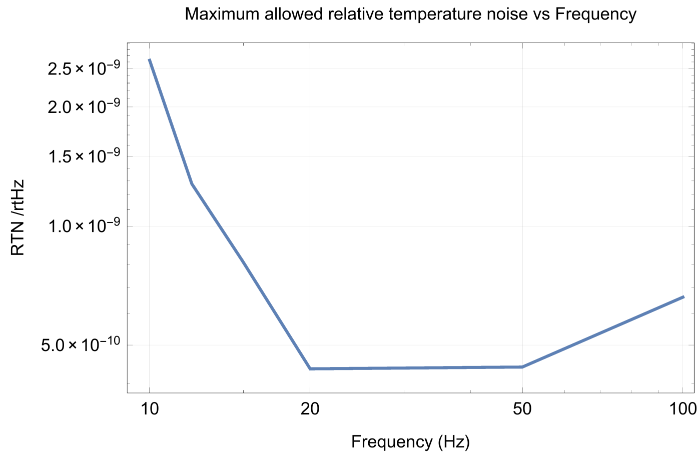
Out[143]=

$$\sqrt{\frac{3.32495\times 10^{-19}}{f^4}+\frac{3.29563\times 10^{-20}}{f^2}}$$

In[204]:=
```
MaxRTN = Table[{MaxDisplacementNoise[[ii]][[1]],
      MaxDisplacementNoise[[ii]][[2]] / (RTNCoefficient /. f → MaxDisplacementNoise[[ii]][[1]])},
   {ii, Length[MaxDisplacementNoise]}];
```

```
In[206]:= ListLogLogPlot[MaxRTN, GridLines → Automatic, Frame → True,
  Joined → True, FrameLabel → {"Frequency (Hz)", "RTN /rtHz",
    "Maximum allowed relative temperature noise vs Frequency"}]
```

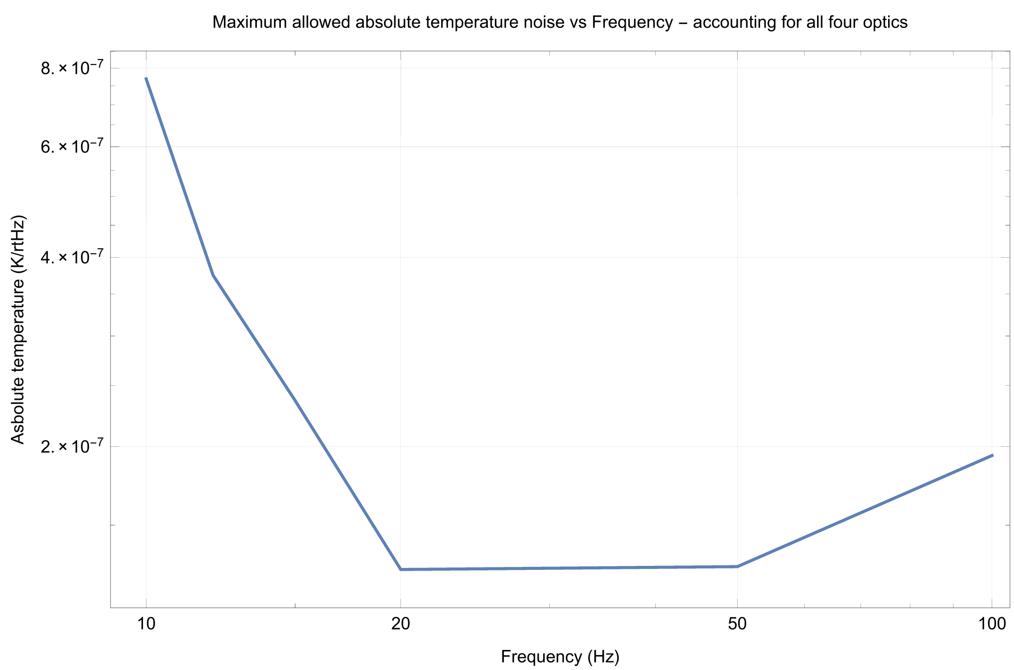
Out[206]=




```
In[209]:= MaxAbsoluteTN = Table[{MaxDisplacementNoise[[ii]][[1]],
       Tenv MaxDisplacementNoise[[ii]][[2]]
  ─────────────────────────────────────────────────────},
   RTNCoefficient /. f → MaxDisplacementNoise[[ii]][[1]]
    {ii, Length[MaxDisplacementNoise]}];
```

```
In[212]:= ListLogLogPlot[MaxAbsoluteTN, GridLines → Automatic,
  Frame → True, Joined → True, FrameLabel →
   {"Frequency (Hz)", "Asbolute temperature (K/rtHz)", "Maximum allowed absolute
      temperature noise vs Frequency - accounting for all four optics"}]
```

Out[212]=



Possible temperature noise?

In[216]:=
```
CurrentLHODisplacementNoise = N[{{10, 2 * 10^-16}, {20, 8 * 10^-18},
    {30, 10^-18}, {40, 3 * 10^-19}, {50, 1.8 * 10^-19}, {60, 10^-19}}];
PossibleAbsoluteTN = Table[{CurrentLHODisplacementNoise[[ii]][[1]],
      Tenv CurrentLHODisplacementNoise[[ii]][[2]] / (RTNCoefficient /. f → CurrentLHODisplacementNoise[[ii]][[1]])},
   {ii, Length[MaxDisplacementNoise]}];
ListLogLogPlot[PossibleAbsoluteTN, GridLines → Automatic,
 Frame → True, Joined → True, FrameLabel →
  {"Frequency (Hz)", "Asbolute temperature (K/rtHz)", "If absolute temperature
    noise were responsible vs Frequency - accounting for all four optics"}]
```

Out[218]=