

1 Calibration of side of fringe measurements

$$P_{trans} = \frac{P_{in}}{1 + [(2\mathcal{F}/\pi) \sin(2\pi x/\lambda)]^2} \quad (1)$$

$$x_{offset} = \frac{\lambda}{2\mathcal{F}} \arcsin \left[\frac{\pi}{2\mathcal{F}} \sqrt{\frac{P_{in}}{P_{trans}} - 1} \right] \quad (2)$$

The transmitted power when the OMC is locked on resonance is P_{in} , and \bar{P}_{trans} is the average transmitted power when locked with an offset x_{offset} .

When the OMC is locked on the side of the fringe, the transmitted power becomes linearly proportional to the length:

$$\frac{dP_{trans}}{dx} = \frac{-P_{trans}^2}{P_{in}} \frac{8\mathcal{F}^2}{\pi\lambda} \sin 4\pi x_{offset}/\lambda \quad (3)$$

2 Dither locking optical gain

When the OMC is locked on resonance with small detunings and a dither line, the transmitted RIN is:

$$\begin{aligned} RIN_{trans} &= \frac{1}{1 + [4\mathcal{F}(\delta x + x_d \sin(\omega_d t))/\lambda]^2} \\ &= \frac{4\mathcal{F}^2}{\lambda} [\delta x^2 + 2\delta x x_d \sin(\omega_d t) + x_d^2] \end{aligned} \quad (4)$$

x_d is the amplitude of the dither line, ω_d the frequency. After demodulation by $A_d \cos(\omega_d)$, the error signal is given by:

$$cnts = \frac{4\mathcal{F}^2}{\lambda} A_d x_d \delta x \quad (5)$$

3 Measured error signal to actual length fluctuations algebra

The PD RIN is low pass filtered with a 100Hz low pass filter F before demodulation, causing the knee in the measured spectra. The real motion of the OMC length with the loop on is given by:

$$r = \frac{x}{1-G} + \frac{nG}{1-G} \quad (6)$$

$$(7)$$

if n is sensor noise and x is the unsuppressed length fluctuations. The measured error signal is:

$$m = \frac{F}{1-G}(x+n) \quad (8)$$

so at frequencies where the error signal is dominated by sensor noise we can relate the measured error signal to the actual motion of the OMC using:

$$r = m \frac{G}{F} \quad (9)$$

And where we think the measured error signal is dominated by residual length fluctuations we can use:

$$r = \frac{m}{F} \quad (10)$$

Because the error signal is only limited by residual length fluctuations at frequencies well below the bandpass F , I did not correct for this.