Length feedforward calculations, LIGO-T1600504

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1 Calculation of first filter

Detailed calculations of length sensing and control (LSC) feedforward have previously been reported in VIR-050A-08, but here I will re-calculate the filters required for Advanced LIGO, as opposed to Initial Virgo. Figure 1 here is equivalent to Figure 1 of the Virgo document, with the exception that we actuate not on the ETMs but on the ITMs differentially. The example here is for the MICH length degree of freedom as noted by the subscript 'M' in variables, but is applicable to any length degree of freedom.

Triangles denoted by 'F' in Figure 1 represent the frequency dependent digital control filters for both DARM and MICH. Squares denoted by 'P' represent the frequency dependent physical plants, including the actuation, optical plant and any analog electronics. Diamonds denoted by 'S' represent frequency-independent sensors. Any frequency dependence is included in 'P'. $P_{(\mathrm{M}\rightarrow\mathrm{S_D})}$ represents the coupling of MICH actuation to DARM sensing, while $P_{(\alpha\rightarrow S_{\mathrm{D}})}$ represents the actuation from the output of the feedforward filter α to DARM. MICH actuation noises are represented by n_{act}, while MICH sensing noises are represented by n_{sen}.

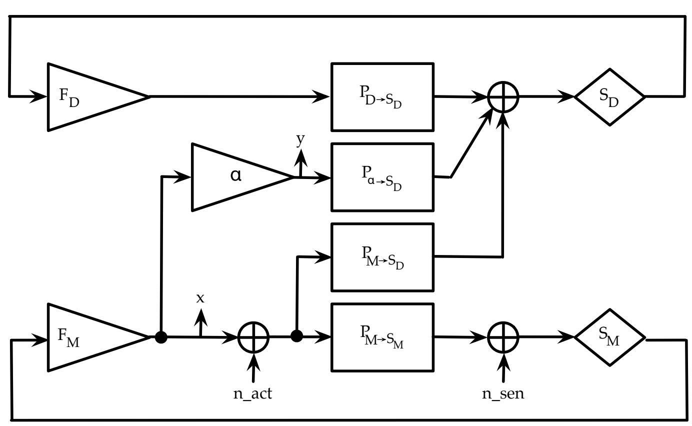

Figure 1: Loop diagram for calculation of first LSC feedforward filter.

The goal of the LSC feedforward is to remove the effect of n_{act} and n_{sen} in the DARM sensor S_{D}. However, the sensing noise will dominate over the actuator noise, so in the following calculations we will ignore n_{act}, although it can be added back into the calculations if desired.

First we will calculate the noise at point x, in the absence of any feedforward, i.e. $\alpha = 0$. We write the usual equation for determining the noise at x in the presence of the closed-loop MICH loop in Equation 1, and solve for x in Equation 2.

$$x = (xP_{(\mathrm{M}\to\mathrm{S_M})} + n_{\mathrm{sen}})F_{\mathrm{M}} \tag{1}$$

$$x = n_{\mathrm{sen}}F_{\mathrm{M}}\left(\frac{1}{1-P_{(\mathrm{M}\to\mathrm{S_M})}F_{\mathrm{M}}}\right) \tag{2}$$

We then write the equation for noise at the DARM sensor, in the presence of the DARM loop, coupling from MICH, and feedforward from MICH. This will allow us to determine the optimal feedforward filter α. Writing S_{D} in the presence of these loops gives

$$S_{\mathrm{D}} = S_{\mathrm{D}}P_{(\mathrm{D}\to\mathrm{S_D})}F_{\mathrm{D}} + xP_{(\mathrm{M}\to\mathrm{S_D})} + x\alpha P_{(\alpha\to\mathrm{S_D})}. \tag{3}$$

or

$$S_{\mathrm{D}} = \left(\frac{1}{1-P_{(\mathrm{D}\to\mathrm{S_D})}F_{\mathrm{D}}}\right)\left(P_{(\mathrm{M}\to\mathrm{S_D})} + \alpha P_{(\alpha\to\mathrm{S_D})}\right)x. \tag{4}$$

Note that here we have assumed that n_{act} is non-existant, and that the input to α and $P_{(\mathrm{M}\to\mathrm{S_D})}$ are identical. Substituting in Equation 2 gives

$$S_{\mathrm{D}} = \left(\frac{1}{1-P_{(\mathrm{D}\to\mathrm{S_D})}F_{\mathrm{D}}}\right)\left(P_{(\mathrm{M}\to\mathrm{S_D})} + \alpha P_{(\alpha\to\mathrm{S_D})}\right)n_{\mathrm{sen}}F_{\mathrm{M}}\left(\frac{1}{1-P_{(\mathrm{M}\to\mathrm{S_M})}F_{\mathrm{M}}}\right). \tag{5}$$

We remove n_{sen} from S_{D} by setting

$$P_{(\mathrm{M}\to\mathrm{S_D})} + \alpha P_{(\alpha\to\mathrm{S_D})} = 0. \tag{6}$$

Solving for α,

$$\alpha = -\frac{P_{(\mathrm{M}\to\mathrm{S_D})}}{P_{(\alpha\to\mathrm{S_D})}}. \tag{7}$$

Intuitively this makes sense as measuring the coupling from MICH to DARM, and dividing out the actuation transfer function.

We note that measuring the transfer functions required for determining α is easier to do when $\alpha = 0$. We calculate the signal at the DARM sensor S_{D}, including the coupling from MICH to DARM, $P_{\mathrm{M}\to\mathrm{S_D}}$. Since the MICH sensor noise is included in the noise at x, we will not write n_{sen} explicitly in Equation 8

$$S_{\mathrm{D}} = S_{\mathrm{D}}P_{(\mathrm{D}\to\mathrm{S_D})}F_{\mathrm{D}} + xP_{(\mathrm{M}\to\mathrm{S_D})}. \tag{8}$$

Solving for S_{D} gives

$$S_{\mathrm{D}} = xP_{(\mathrm{M}\rightarrow\mathrm{S_D})}\left(\frac{1}{1 - P_{(\mathrm{D}\rightarrow\mathrm{S_D})}F_{\mathrm{D}}}\right). \tag{9}$$

We will eventually need to measure the transfer function $\frac{S_{\mathrm{D}}}{x}$ from MICH to DARM, so we write that explicitly in Equation 10,

$$\frac{S_{\mathrm{D}}}{x} = P_{(\mathrm{M}\rightarrow\mathrm{S_D})}\left(\frac{1}{1 - P_{(\mathrm{D}\rightarrow\mathrm{S_D})}F_{\mathrm{D}}}\right). \tag{10}$$

To determine $P_{(\alpha\rightarrow\mathrm{S_D})}$, we will need to measure the transfer function $\frac{S_{\mathrm{D}}}{y}$, so we write S_{D}'s response similar to Equation 8

$$S_{\mathrm{D}} = S_{\mathrm{D}}P_{(\mathrm{D}\rightarrow\mathrm{S_D})}F_{\mathrm{D}} + yP_{(\alpha\rightarrow\mathrm{S_D})}. \tag{11}$$

Solving for $\frac{S_{\mathrm{D}}}{y}$ gives

$$\frac{S_{\mathrm{D}}}{y} = P_{(\alpha\rightarrow\mathrm{S_D})}\left(\frac{1}{1 - P_{(\mathrm{D}\rightarrow\mathrm{S_D})}F_{\mathrm{D}}}\right). \tag{12}$$

Finally, we note that the ratio of the transfer functions in Equation 10 and Equation 12 is equal to $-\alpha$

$$\alpha = -\frac{\left(\frac{S_{\mathrm{D}}}{x}\right)}{\left(\frac{S_{\mathrm{D}}}{y}\right)}. \tag{13}$$

In practice, $\frac{S_{\mathrm{D}}}{x}$ is measured from MICH OUT to DARM IN. We do not have excitation points at the output of the filter banks, so we drive at MICH IN.

Once we have calculated a frequency-dependent α, we must use a fitting program such as Vectfit to extract a model for the filter that can be utilized in LIGO's front end system. We will often also add elements to the filter path such as AC-coupling via a high-pass filter, a high frequency roll-off filter, and notches so as not to excite fundamental modes of the optics being driven.

2 Calculation of iterative filters

Often, the fit to the calculated filter will not be perfect, or frequencies with low measured coherence will affect the fit. Also, extra elements such as AC-coupling and roll-off filters will affect the phase of the feedforward filter in the band of interest. For this reason, it is useful to be able to iteratively update the feedforward filter. However, this must be done with the existing feedforward turned on, so the loop calculations are somewhat more complicated. Figure 2 is very similar to Figure 1, but has the iteratively updateable portion of the feedforward filter explicitly separated into a series filter β. The point x in Figure 1 has been renamed z in Figure 2 for clarity. Measured transfer functions $\frac{S_{\mathrm{D}}}{x}$ and $\frac{S_{\mathrm{D}}}{y}$ imply the use of results from the calculation of the original filter

α, while measured transfer function $\frac{S_{\rm D}}{z}$ will be a fresh measurement for the calculation of β. Note that α here must represent the actual filter in use for the feedforward system, such that it includes any extra elements such as the high- and low-passing.

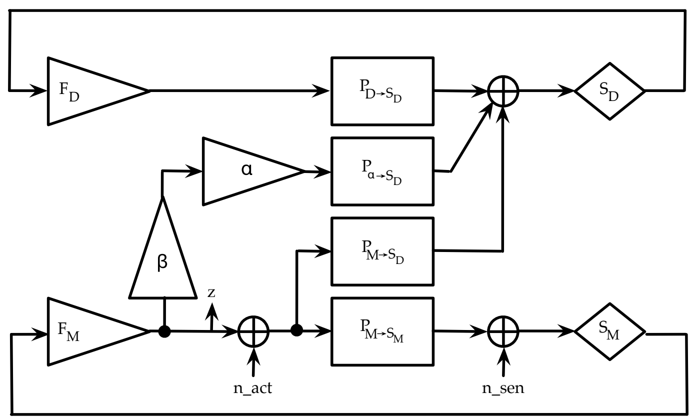

Figure 2: Loop diagram for calculation of iterative LSC feedforward filter.

First we will calculate the noise at $S_{\rm D}$, equivalent to Equation 3

$$S_{\rm D} = S_{\rm D} P_{\rm (D \to S_D)} F_{\rm D} + x P_{\rm (M \to S_D)} + z\alpha\beta P_{(\alpha \to {\rm S_D})}. \tag{14}$$

Solving for $S_{\rm D}$ and recognizing that $z \propto n_{\rm sen}$ just as $x \propto n_{\rm sen}$ in Equation 2, we find

$$S_{\rm D} = \left(\frac{1}{1 - P_{\rm (D \to S_D)} F_{\rm D}}\right) \left(P_{\rm (M \to S_D)} + \alpha\beta P_{(\alpha \to {\rm S_D})}\right) n_{\rm sen} F_{\rm M} \left(\frac{1}{1 - P_{\rm (M \to S_M)} F_{\rm M}}\right). \tag{15}$$

Eliminating the effect of $n_{\rm sen}$ on $S_{\rm D}$, we will set

$$P_{\rm (M \to S_D)} + \alpha\beta P_{(\alpha \to {\rm S_D})} = 0. \tag{16}$$

This gives

$$\beta = -\frac{P_{\rm (M \to S_D)}}{\alpha P_{(\alpha \to {\rm S_D})}}. \tag{17}$$

In order to measure the required transfer functions for Equation 17, we will calculate $\frac{S_{\rm D}}{z}$ with β set to 1. This is an extension of Equation 8:

$$S_{\rm D} = S_{\rm D} P_{\rm (D \to S_D)} F_{\rm D} + z P_{\rm (M \to S_D)} + z\alpha P_{(\alpha \to {\rm S_D})}. \tag{18}$$

Solving for $\frac{S_{\rm D}}{z}$ gives

$$\frac{S_{\rm D}}{z} = \frac{P_{\rm (M \to S_D)} + \alpha P_{(\alpha \to {\rm S_D})}}{1 - P_{\rm (D \to S_D)} F_{\rm D}}. \tag{19}$$

We solve Equation 19 for $P_{(\mathrm{M}\to\mathrm{S_D})}$, which we will insert into Equation 17,

$$P_{(\mathrm{M}\to\mathrm{S_D})} = \left(\frac{S_\mathrm{D}}{z}\right)\left(1 - P_{(\mathrm{D}\to\mathrm{S_D})}F_\mathrm{D}\right) - \alpha P_{(\alpha\to\mathrm{S_D})}. \tag{20}$$

Similarly, we need to solve Equation 12 for $P_{(\alpha\to\mathrm{S_D})}$ to insert into Equation 17,

$$P_{(\alpha\to\mathrm{S_D})} = \left(\frac{S_\mathrm{D}}{y}\right)\left(1 - P_{(\mathrm{D}\to\mathrm{S_D})}F_\mathrm{D}\right). \tag{21}$$

Plugging Equation 20 into Equation 17, we find

$$\beta = -\frac{\left(\frac{S_\mathrm{D}}{z}\right)\left(1 - P_{(\mathrm{D}\to\mathrm{S_D})}F_\mathrm{D}\right) - \alpha P_{(\alpha\to\mathrm{S_D})}}{\alpha P_{(\alpha\to\mathrm{S_D})}} \tag{22}$$

$$\beta = 1 - \frac{\left(\frac{S_\mathrm{D}}{z}\right)\left(1 - P_{(\mathrm{D}\to\mathrm{S_D})}F_\mathrm{D}\right)}{\alpha P_{(\alpha\to\mathrm{S_D})}}. \tag{23}$$

Inserting Equation 21 gives

$$\beta = 1 - \frac{\left(\frac{S_\mathrm{D}}{z}\right)\left(1 - P_{(\mathrm{D}\to\mathrm{S_D})}F_\mathrm{D}\right)}{\alpha\left(\frac{S_\mathrm{D}}{y}\right)\left(1 - P_{(\mathrm{D}\to\mathrm{S_D})}F_\mathrm{D}\right)} \tag{24}$$

$$\beta = 1 - \frac{\left(\frac{S_\mathrm{D}}{z}\right)}{\alpha\left(\frac{S_\mathrm{D}}{y}\right)} \tag{25}$$

This requires knowledge of the "actuator" portion of the initial α calculation, from the transfer function $\frac{S_\mathrm{D}}{y}$, the pre-existing feedforward filters, and a fresh measurement of the coupling by measuring between MICH OUT and DARM IN. Equation 25 should converge to a flat response of 1 when there is no more coupling of MICH noise into the DARM sensor, i.e. when $\frac{S_\mathrm{D}}{z}$ is very small.

This iterative procedure can be repeated as many times as necessary. For any future iterations, the β calculated here is subsumed into α as a pre-existing filter, and a new β can be found.