



FIGURE 19.18

Geometry for analyzing misalignment and axis displacement in a stable optical resonator.

misalignments is to use the techniques for misaligned ray matrix systems discussed in Section 15.4. We can give in this section, however, a brief description of the axis displacement and misalignment produced in a simple two-mirror cavity by angular misalignment of either end mirror.

Misalignment Analysis

The optical axis in a two-mirror resonator is by definition the line passing through the centers of curvature C_1 and C_2 of the two end mirrors. The quadratic phase curvatures of the two mirrors are centered on or normal to this axis. If the cavity also contains any kind of aperture (including the apertures defined by the mirrors themselves), rotation of an end mirror will translate the optical axis relative to this aperture or, alternatively, will cause the aperture to be effectively off-center with respect to the resonator axis. The presence of an off-center aperture will tend to produce resonator eigenmodes which are mixtures of the even and odd eigenmodes of the aligned resonator. Solving for the exact eigenmodes and their exact diffraction losses in this situation becomes a complicated calculation.

Simple geometry can at least tell us how far the optical axis will be translated and rotated by a small angular rotation of either end mirror. Let θ_1 and θ_2 be the small angular rotations of the two end mirrors and Δx_1 and Δx_2 be the small sideways translations of the new or misaligned optical axis at the point where it intercepts the end mirrors, as shown in Figure 19.18. (Alternatively, Δx_1 and Δx_2 can represent the off-center translations of the apertures at those two mirrors.) From Figure 19.18 and some simple geometry, we can then evaluate these displacements as

$$\begin{aligned}\Delta x_1 &= \frac{g_2}{1 - g_1 g_2} \times L \theta_1 + \frac{1}{1 - g_1 g_2} \times L \theta_2 \\ \Delta x_2 &= \frac{1}{1 - g_1 g_2} L \Delta \theta_1 + \frac{g_1}{1 - g_1 g_2} L \Delta \theta_2.\end{aligned}\quad (32)$$

One criterion for judging the seriousness of misalignment effects is then to compare these displacements Δx_1 and Δx_2 with the resonator spot sizes w_1 and w_2 at the same end mirrors.

The angular displacement of the resonator axis (which can be important in evaluating far-field pointing accuracy, for example) can also be evaluated from

$$\Delta \theta \equiv \frac{\Delta x_2 - \Delta x_1}{L} = \frac{(1 - g_2) \theta_1 - (1 - g_1) \theta_2}{1 - g_1 g_2}. \quad (33)$$

Note that the sensitivity of all these measures to angular misalignment blows up as $g_1 g_2 \rightarrow 1$, i.e., as the resonator design approaches the stability boundary on either the planar (long-radius) or the near-concentric sides of the stability region.

REFERENCES

Misalignment effects in stable resonators are treated in more detail, and with supporting experimental results, by R. Hauck, H. P. Kortz, and H. Weber, "Misalignment sensitivity of optical resonators," *Appl. Optics* **19**, 598-601 (February 15, 1980).

Exact calculations of the effects of mirror tilt on resonator losses for planar resonators, with both strip and circular mirrors, are given by J. L. Remo, "Diffraction losses for symmetrically tilted plane reflectors in open resonators," *Appl. Optics* **19**, 774-777 (March 1, 1980).

19.5 GAUSSIAN RESONATOR MODE LOSSES

The gaussian beam results developed in this chapter thus far are based on the assumption that the resonator end mirrors are infinitely wide in the transverse direction, or at least extend out so far compared to the gaussian spot size of the gaussian modes that any aperture diffraction effects are entirely negligible.

Introduction of a finite aperture into a stable gaussian resonator then modifies these results, though generally by a small amount if the aperture diameter is large compared to the gaussian spot size. In this section we will review briefly the mode distortions and diffraction losses that result from introduction of finite apertures or mirror sizes.

Resonator Fresnel Number

A very important parameter for discussing aperture effects in finite-diameter stable (or for that matter unstable) optical resonators is the *resonator Fresnel number* N_f , which is commonly defined as follows. Let $2a$ represent the transverse width of the resonator end mirrors in the x or y directions in a one-dimensional strip mirror situation, or alternatively the diameter of the circular end mirrors in a circularly symmetric situation. The resonator Fresnel number N_f is then defined, just as in the previous chapter, by

$$\text{resonator Fresnel number, } N_f \equiv \frac{a^2}{L\lambda}. \quad (34)$$

This parameter is obviously the number of Fresnel zones across one end mirror, as seen from the center of the opposite mirror. There are also, however, a number of other significant interpretations of this parameter.