SRCL Dither to Arm Power Measurement

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Using radiation pressure coupling to DARM, we can extract the the average power in the arms by dithering SRCL. This is an overview of the coupling mechanism.

1 SRCL and Arm Power Definitions

From [1], Equation (14) and (15):

$$\begin{aligned}\frac{P_x(f)}{l_s(f)} &= \frac{8g_s^2 r_s r_a' \epsilon k}{t_s^2(1+s_{rse})} P_x \qquad (1)\\ &= \gamma P_x \qquad (2)\end{aligned}$$

where P_x is the power in the X arm, l_s is the signal recycling cavity length, g_s is the amplitude signal recycling cavity gain, r_s is the SRM amplitude reflectivity, r_a' is the arm reflectivity derivative with respect to phase, k is the laser wavenumber, t_s is the SRM amplitude transmission, and $s_{rse} = i\omega/\omega_{rse}$ is the resonant signal extraction (rse) DARM cavity pole. For this measurement, I gather the optical response of the SRCL dither into a factor γ. The power in the Y arm is the same except for an overall sign flip:

$$\begin{aligned}\frac{P_y(f)}{l_s(f)} &= -\frac{8g_s^2 r_s r_a' \epsilon k}{t_s^2(1+s_{rse})} P_y \qquad (3)\\ &= -\gamma P_y \qquad (4)\end{aligned}$$

Now, the actual measurement I can perform is from SRCL_OUT to TRX and TRY, so I define a few more factors.

Compliance of the SRM suspension:

$$\frac{l_s(f)}{F_s(f)} = -\frac{1}{m\omega^2} \qquad (5)$$

the compliance of the SRM, where F_s is the force applied to the SRM, and m is the SRM mass, and ω is the audio frequency.

SRCL control signal calibration:

$$\frac{F_s(f)}{c_s(f)} = \beta \left[\frac{\mathrm{N}}{\mathrm{cts}}\right] \qquad (6)$$

where c_s is the SRCL control signal in counts and β is just some calibration.

Transmitted power:

$$\frac{P_{tx1}}{P_x} = T_{ex}\eta_{x1} \tag{7}$$

where P_{tx1} is the transmitted power through the X arm falling on the X_TR_A photodiode, T_{ex} is the power transmission through ETMX, and η_{x1} is the loss/response/calibration of the X_TR_A PD. There is a X_TR_B PD which I call x2, and a Y_TR_A and Y_TR_B for y1, y2.

Finally, we have relative intensity:

$$RIN(f) = \frac{P(f)}{\bar{P}} \tag{8}$$

2 Contructing TRX/SRCL, TRY/SRCL

I have measured the transfer functions from SRCL_OUT to {X,Y}_TR_{A,B}. In principle, these measurements should reflect the following:

$$\begin{align}
\frac{P_{tx1}(f)}{c_s(f)} &= \frac{F_s(f)}{c_s(f)}\frac{l_s(f)}{F_s(f)}\frac{P_x(f)}{l_s(f)}\frac{P_{tx1}(f)}{P_x(f)} \tag{9}\\
&= \beta \times -\frac{1}{m\omega^2} \times \gamma P_x \times T_{ex}\eta_{x1} \tag{10}\\
&= -\frac{\beta\gamma T_{ex}\eta_{x1}}{m\omega^2} P_x \tag{11}
\end{align}$$

We may divide by the average power on the PD to get the RIN TF and eliminate dependence on the PD calibration, ETM transmission, and arm power:

$$\frac{\left(\dfrac{P_{tx1}(f)}{c_s(f)}\right)}{\bar{P_{tx1}}} = -\frac{\beta\gamma}{m\omega^2} \tag{12}$$

I have four measurements of the above quantity: X_TR_A/SRCL_OUT, X_TR_B/SRCL_OUT, Y_TR_A/SRCL_OUT, Y_TR_B/SRCL_OUT. They agree to 4%.

3 Radiation Pressure Coupling to DARM

From equations 1 and 3 we see the arm power change from the SRCL dither is differential. The change in arm power will change the radiation pressure force, and arm lengths.

DARM

$$L_{DARM} = \frac{L_x - L_y}{2} \tag{13}$$

where L_x and L_y are the X and Y arm lengths.

Radiation Pressure Force

$$F_i(f) = \frac{2P(f)}{c} \tag{14}$$

where $F_i(f)$ is the force on a single optic, and $P(f)$ is the power in the arm.

Quad Force to Length

$$L_i(f) = -\frac{F_i(f)}{m_q \omega^2} \tag{15}$$

where $L_i(f)$ is the displacement of a single optic, and m_q is the mass of the quad.

Combining Eqs. 14 and 15, and multiplying by two for both the ETM and ITM:

$$L_x(f) = -\frac{4P_x(f)}{m_q c \omega^2} \tag{16}$$

or, for DARM,

$$L_{DARM}(f) = -\frac{4P_{arm}(f)}{m_q c \omega^2} \tag{17}$$

Again, bringing in Eqs 1 and 3, we can find the SRCL to DARM radiation pressure coupling:

$$\frac{L_{DARM}(f)}{l_s(f)} = -\frac{4\gamma P_{arm}}{m_q c \omega^2} \tag{18}$$

or, looking at the SRCL control signal from Eqs 5 and 6:

$$\frac{L_{DARM}(f)}{c_s(f)} = -\frac{4\beta\gamma P_{arm}}{m_q m c \omega^4} \tag{19}$$

4 Combining Measurements

I calculated the arm power by fitting some constants α_1, α_2 and comparing Eqs 19 and 12:

$$\frac{\left(\frac{L_{DARM}(f)}{c_s(f)}\right)}{\frac{\left(\frac{P_{tx1}(f)}{c_s(f)}\right)}{\bar{P}_{tx1}}} = \frac{\left(\frac{\alpha_1}{f^4}\right)}{\left(\frac{\alpha_2}{f^2}\right)} \tag{20}$$

$$= \frac{4P_{arm}}{m_q c \omega^2} \tag{21}$$

$$= \frac{P_{arm}}{m_q c \pi^2 f^2} \tag{22}$$

$$\implies P_{arm} = \frac{\alpha_1}{\alpha_2}\pi^2 m_q c \tag{23}$$

A SRCL dither measurement was taken at 30 watts input power. With a power recycling gain of 43, and an arm power gain of about 280, we would expect 180 kW in each arm. $m_q = 40\text{kg}$, $\alpha_1 = 7.11 \pm 0.07 \times 10^{-13}$. $\alpha_2 = 6.6 \pm 0.3 \times 10^{-7}$,

$$P_{arm} = 127.2 \pm 5.4\text{kW} \tag{24}$$

P_{arm} represents the average power in the arms. From HEPI offloading during INCREASE_POWER, I find the ratio of arm powers to be

$$\frac{P_x}{P_y} = 0.9 \pm 0.1 \tag{25}$$

Which gives for each arm

$$P_x = 121.6 \pm 13.8\text{kW} \tag{26}$$

$$P_y = 132.8 \pm 15.1\text{kW} \tag{27}$$

Future, more clever people might just taken the TF from the arm power transmission to DARM directly while dithering SRCL.

References

[1] K. Izumi, D. Sigg, *Frequency Response of the aLIGO Interferometer: part3* `https://dcc.ligo.org/LIGO-T1500559`