Effective open loop transfer functions for the SOFT/HARD modes with actuation and sensing imbalance

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January 16, 2019

1 Actuation imbalance

Consider a single arm case. We have

$$\begin{bmatrix}\theta_h\\ \theta_s\end{bmatrix} = \begin{bmatrix}P_h & 0\\ 0 & P_s\end{bmatrix}\begin{bmatrix}\tau_h\\ \tau_s\end{bmatrix}, \qquad \begin{bmatrix}\tau_h\\ \tau_s\end{bmatrix} = \begin{bmatrix}P_h^{-1} & 0\\ 0 & P_s^{-1}\end{bmatrix}\begin{bmatrix}\theta_h\\ \theta_s\end{bmatrix}, \tag{1}$$

where we have used the τ's for actuation torques, θ's for angular response, and P for suspension plant.

In an ideal case and restricted to the pitch torques, for aLIGO geometry we have

$$\begin{bmatrix}\tau_h\\ \tau_s\end{bmatrix} = \begin{bmatrix}1 & 0.87\\ 0.87 & 1\end{bmatrix}\begin{bmatrix}\tau_e\\ \tau_i\end{bmatrix}, \tag{2}$$

where in the subscripts $h(s)$ stands for HARD (SOFT) and $e(i)$ for ETM (ITM). This can be inverted to get

$$\begin{bmatrix}\tau_e\\ \tau_i\end{bmatrix} = \begin{bmatrix}4.1 & -3.6\\ -3.6 & 4.1\end{bmatrix}\begin{bmatrix}\tau_h\\ \tau_s\end{bmatrix}. \tag{3}$$

From now on, we write our actuations as vectors in basis of the ideal hard and soft modes, $[\tau_h, \tau_s]$. E.g., $\vec{\tau}_h = [1, 0][\tau_h, \tau_s]^T$ and $\vec{\tau}_e = [4.1, -3.6][\tau_h, \tau_s]^T$. Thus the $\delta\vec{\tau}_e$ means a drive that goes to both the HARD and SOFT actuations, while its magnitude is labeled by $\delta\tau_e$ (i.e., no vector symbol). We further let $\delta\tau_e$ to be a dimensionless number of order unity or smaller, and let $\tau_{h,(s)}$ to absorb the physical dimensions. In the following we will use vectors $\vec{\tau}_{h,(s)}$ and $\vec{\theta}_{h,(s)}$ to denote torque and angle corresponds to the ideal hard/soft modes, and $\vec{\tau}'_{h,(s)}$ and $\vec{\theta}'_{h,(s)}$ in the presence of actuation and sensing imbalances.

In the presence of actuation imperfections, a hard mode drive can be written as

$$
\begin{aligned}
\vec{\tau}_h' &= [1,\ 0.87] \begin{bmatrix} \vec{\tau}_e + \delta\vec{\tau}_e \\ \vec{\tau}_i + \delta\vec{\tau}_i \end{bmatrix}, \\
&= [1, 0] \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix} + [1, 0.87] \begin{bmatrix} \tau_e & 0 \\ 0 & \tau_i \end{bmatrix} \begin{bmatrix} 4.1 & -3.6 \\ -3.6 & 4.1 \end{bmatrix} \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix}, \\
&= \vec{\tau}_h + [4.1\delta\tau_e - 3.1\delta\tau_i, -3.6\delta\tau_e + 3.6\delta\tau_i] \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix}. \qquad (4)
\end{aligned}
$$

Similarly, a soft drive corresponds to

$$
\begin{aligned}
\vec{\tau}_s' &= [0.87,\ 1] \begin{bmatrix} \vec{\tau}_e + \delta\vec{\tau}_e \\ \vec{\tau}_i + \delta\vec{\tau}_i \end{bmatrix}, \\
&= [0, 1] \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix} + [0.87, 1] \begin{bmatrix} \tau_e & 0 \\ 0 & \tau_i \end{bmatrix} \begin{bmatrix} 4.1 & -3.6 \\ -3.6 & 4.1 \end{bmatrix} \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix}, \\
&= \vec{\tau}_s + [3.6\delta\tau_e - 3.6\delta\tau_i, -3.1\delta\tau_e + 4.1\delta\tau_i] \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix}. \qquad (5)
\end{aligned}
$$

Putting together

$$
\begin{bmatrix} \vec{\tau}_h' \\ \vec{\tau}_s' \end{bmatrix} = \begin{bmatrix} \vec{\tau}_h \\ \vec{\tau}_s \end{bmatrix} + \begin{bmatrix} 4.1\delta\tau_e - 3.1\delta\tau_i, & -3.6\delta\tau_e + 3.6\delta\tau_i \\ 3.6\delta\tau_e - 3.6\delta\tau_i, & -3.1\delta\tau_e + 4.1\delta\tau_i \end{bmatrix} \begin{bmatrix} \tau_h \\ \tau_s \end{bmatrix}. \qquad (6)
$$

We want the torque to angle transfer function, which can be obtained by utilizing eq. (1). We have

$$
\begin{aligned}
\begin{bmatrix} \vec{\tau}_h' \\ \vec{\tau}_s' \end{bmatrix} &= \begin{bmatrix} 1 + (4.1\delta\tau_e - 3.1\delta\tau_i), & -3.6\delta\tau_e + 3.6\delta\tau_i \\ 3.6\delta\tau_e - 3.6\delta\tau_i, & 1 + (-3.1\delta\tau_e + 4.1\delta\tau_i) \end{bmatrix} \begin{bmatrix} P_h^{-1} & 0 \\ 0 & P_s^{-1} \end{bmatrix} \begin{bmatrix} \theta_h \\ \theta_s \end{bmatrix} \\
&= \mathbf{\Pi}'^{-1}(f) \begin{bmatrix} \theta_h \\ \theta_s \end{bmatrix}, \qquad (7)
\end{aligned}
$$

Then the inversion of matrix $\mathbf{\Pi}'^{-1}(f)$ gives the effective suspension transfer functions $\mathbf{\Pi}'$ from imbalanced drives $[\vec{\tau}_h', \vec{\tau}_s']$ to the ideal HARD/SOFT angles $[\theta_h, \theta_s]$. Note that $\mathbf{\Pi}'^{-1}$ is frequency dependent, and thus the inversion needs to happen at each frequency bin.

2 Sensor cross coupling

Similar to the actuation imbalance, we can write the detected angle as vectors in the ideal soft/hard basis.

$$\begin{bmatrix} \vec{\theta}'_h \\ \vec{\theta}'_s \end{bmatrix} = \begin{bmatrix} 1 & \delta\theta_{sh} \\ \delta\theta_{hs} & 1 \end{bmatrix} \begin{bmatrix} \theta_h \\ \theta_s \end{bmatrix} \tag{8}$$

$$= \boldsymbol{\Theta}' \begin{bmatrix} \theta_h \\ \theta_s \end{bmatrix}, \tag{9}$$

where again we let $\theta_h and \theta_s$ carry the physical dimension and $\delta\theta_{sh,(hs)}$ be dimensionless quantity characterizing the sensing cross coupling from soft to hard (or hard to soft) modes.

3 Loop effects

The control filters link the angle detected back to actuating torques. We define

$$\begin{bmatrix} \vec{\tau}'_h \\ \vec{\tau}'_s \end{bmatrix} = \begin{bmatrix} K_h & 0 \\ 0 & K_s \end{bmatrix} \begin{bmatrix} \vec{\theta}'_h \\ \vec{\theta}'_s \end{bmatrix} = \boldsymbol{K} \begin{bmatrix} \vec{\theta}'_h \\ \vec{\theta}'_s \end{bmatrix} \tag{10}$$

.

Now the steady-state loop effect can be modeled as

$$\begin{bmatrix} \vec{\tau}'_h \\ \vec{\tau}'_s \end{bmatrix} = -\boldsymbol{K\Theta'\Pi'} \begin{bmatrix} \vec{\tau}'_h \\ \vec{\tau}'_s \end{bmatrix} + \begin{bmatrix} \vec{\tau}'_{h,\mathrm{in}} \\ \vec{\tau}'_{s,\mathrm{in}} \end{bmatrix}, \tag{11}$$

From which we get the close loop response

$$\boldsymbol{G}_{\mathrm{cl}} = \left(\mathbb{1} + \boldsymbol{K\Theta'\Pi'}\right)^{-1}. \tag{12}$$

4 An example

At LHO, the measured DHARD PIT loop exhibits anomalous phase behavior at around 0.8 Hz. We now show that this feature can be reproduced by the model described above.

The effective open loop transfer function in the presence of cross-coupling can be derived from the closed-loop response, as

$$G_{\mathrm{ol},hh} = \frac{1}{G_{\mathrm{cl},hh}} - 1, \tag{13}$$

where $G_{\mathrm{cl},hh}$ is the hard-to-hard component of the closed-loop transfer matrix $\boldsymbol{G}_{\mathrm{cl}}$.

The comparison of the ideal hard mode open loop transfer function, the one with actuation and imperfections, and the measurement data is shown in Fig. 1. We can see that once we include the cross-coupling terms, the model reproduces the phase advance feature in the measured data. To generate the result, the parameters we used are $\delta\tau_e = 0.1$, $\delta\tau_i = 0$, $\delta\theta_{sh} = 0$, and $\delta\theta_{hs} = -1$.

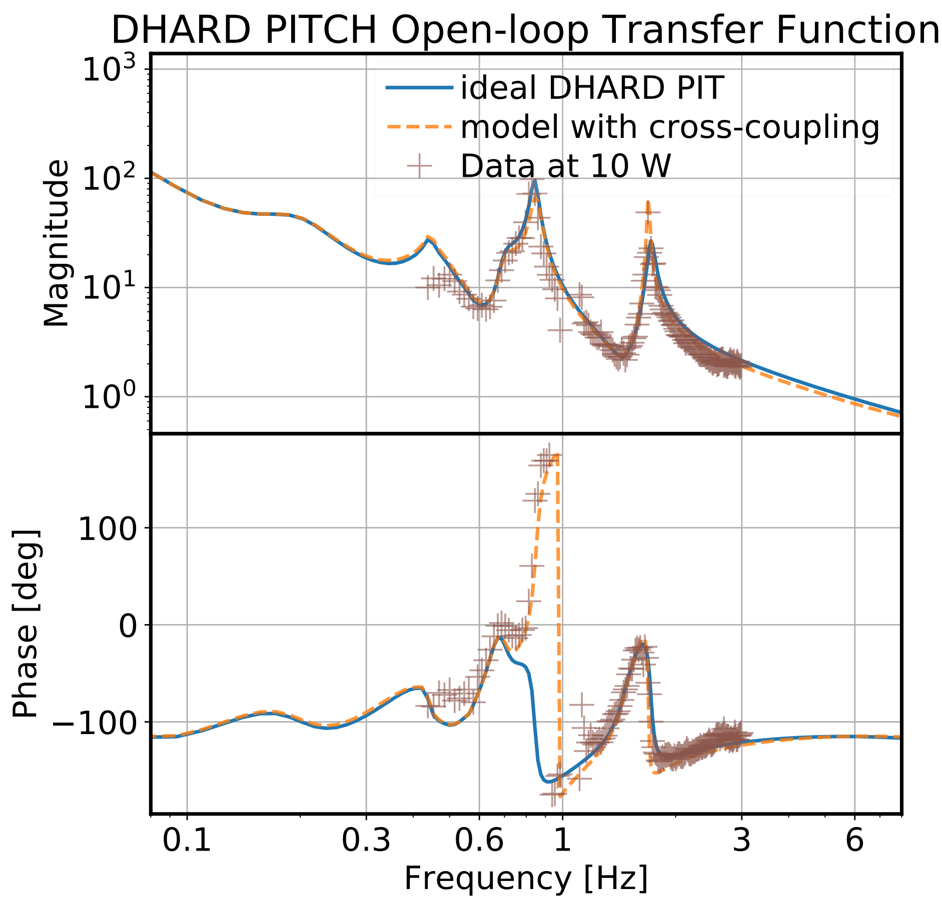

Figure 1: The open loop transfer function of the DHARD pit loop (in the high-bandwidth configuration). The measured data showed an anomalous phase advance at ~ 0.8 Hz relative to the model, which can be well explained once we include 10% actuation imbalance between the ETM and ITM, and a hard-to-soft sensing coupling coefficient $\delta\theta hs = -1$.