DARM bicoherence

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Summary

- What is bicoherence
- DARM bicoherence and what it means
- How do we know it’s not an artifact?
  - Repeatability
  - Numerical experiments

What is bicoherence?

A method to find quadratic relations in the frequency content of a signal

If there is a quadratic process, then frequencies are mixed:

X(f1) and X(f2) mix coherently to produce X(f1+f2) and X(f1-f2)

For reference, here's the usual linear coherence between two signals

$$C_{x,y}(f) = \frac{\left|\langle X^*(f)Y(f)\rangle\right|^2}{\langle X^*(f)X(f)\rangle \langle Y^*(f)Y(f)\rangle}$$

Bicoherence can be defined similarly (for one signal X)

$$bic^2(f_1,f_2) \triangleq \frac{|B(f_1,f_2)|^2}{E[|X(f_1)X(f_2)|^2]\ E[|X(f_1+f_2)|^2]}$$

https://link.springer.com/chapter/10.1007/978-3-540-79224-6_3

Bicoherence has a lot of symmetries in the f1,f2 plane

x = sin(2*pi*7*t)
+ cos(2*pi*3*t)
y = x^2
Plus random uncorrelated noise for both signals

APPENDIX A
SYMMETRIES OF THE BISPECTRUM

From the (6) definition it follows that the bispectrum has several symmetries when the signal is real.

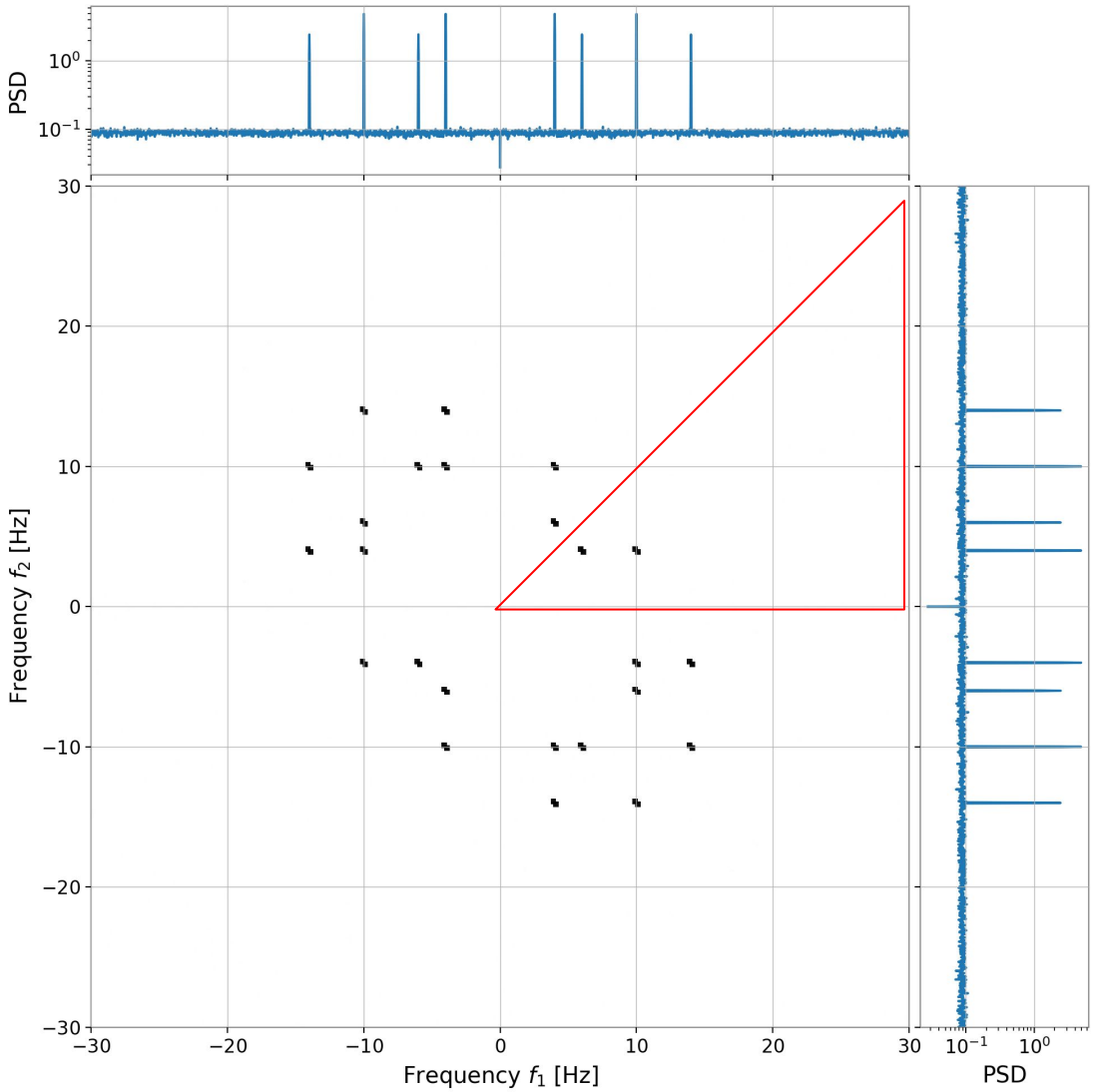
$$\underline{B(f_1,f_2)} = \mathbf{A}\{X(f_1)X(f_2)X^*(f_1+f_2)\} = \mathbf{A}\{X(f_2)X(f_1)X^*(f_2+f_1)\} = \underline{B(f_2,f_1)} \quad (21)$$

$$\underline{B(f_1,f_2)} = \mathbf{A}\{X(f_1)X(f_2)X^*(f_1+f_2)\} = \mathbf{A}\{X^*(f_1)X^*(f_2)X(-f_1-f_2)\} = \underline{B^*(-f_1,-f_2)} \quad (22)$$

$$\underline{B(f_1,f_2)} = \mathbf{A}\{X(f_1)X(f_2)X^*(f_1+f_2)\} = \mathbf{A}\{X(-f_1-f_2)X(f_2)X^*(-f_1)\} = \underline{B(-f_1-f_2,f_2)} \quad (23)$$

$$\underline{B(f_1,f_2)} = \mathbf{A}\{X(f_1)X(f_2)X^*(f_1+f_2)\} = \mathbf{A}\{X(f_1)X(-f_1-f_2)X^*(-f_2)\} = \underline{B(f_1,-f_1-f_2)} \quad (24)$$

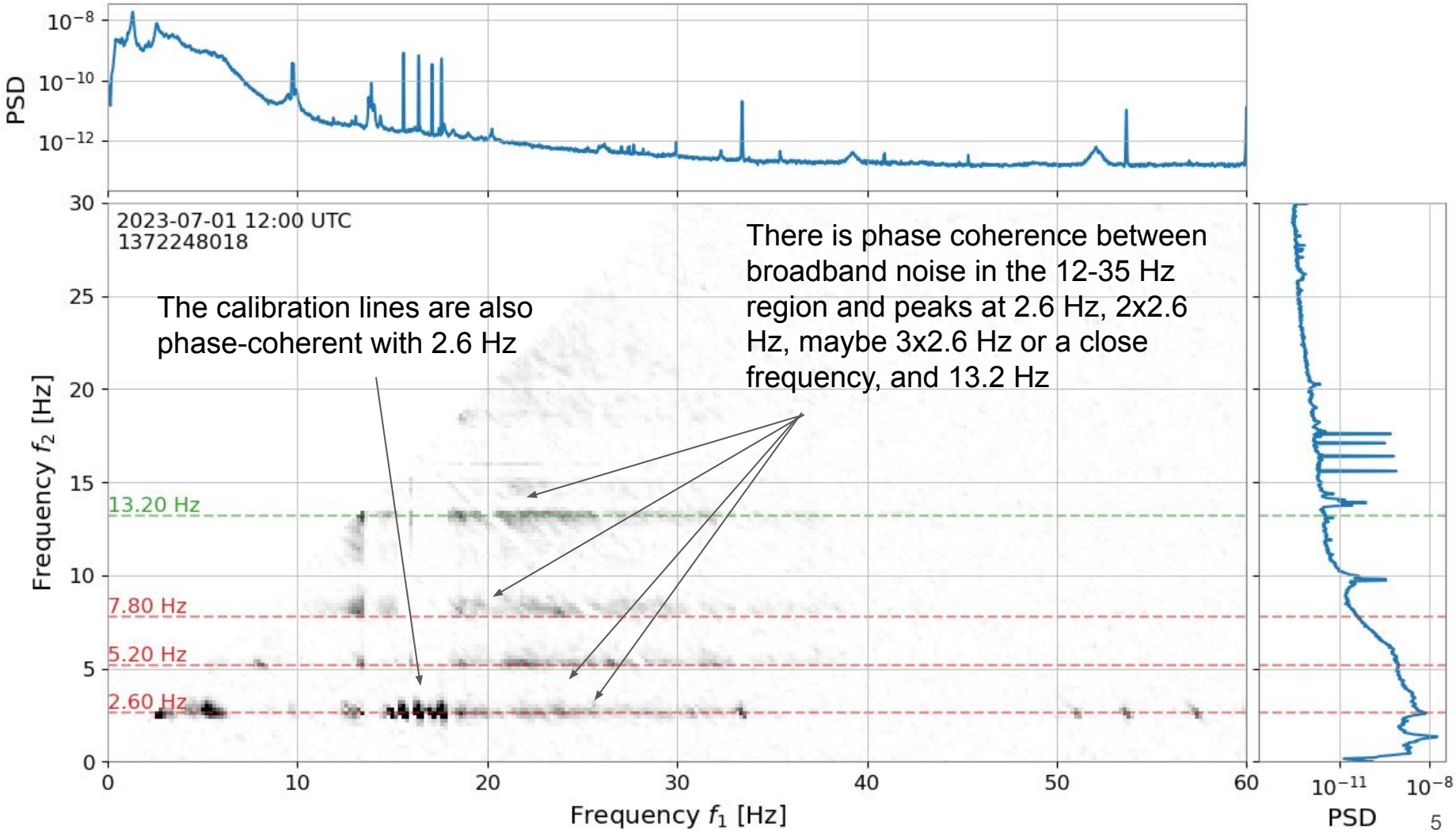
arXiv:1811.02973v1



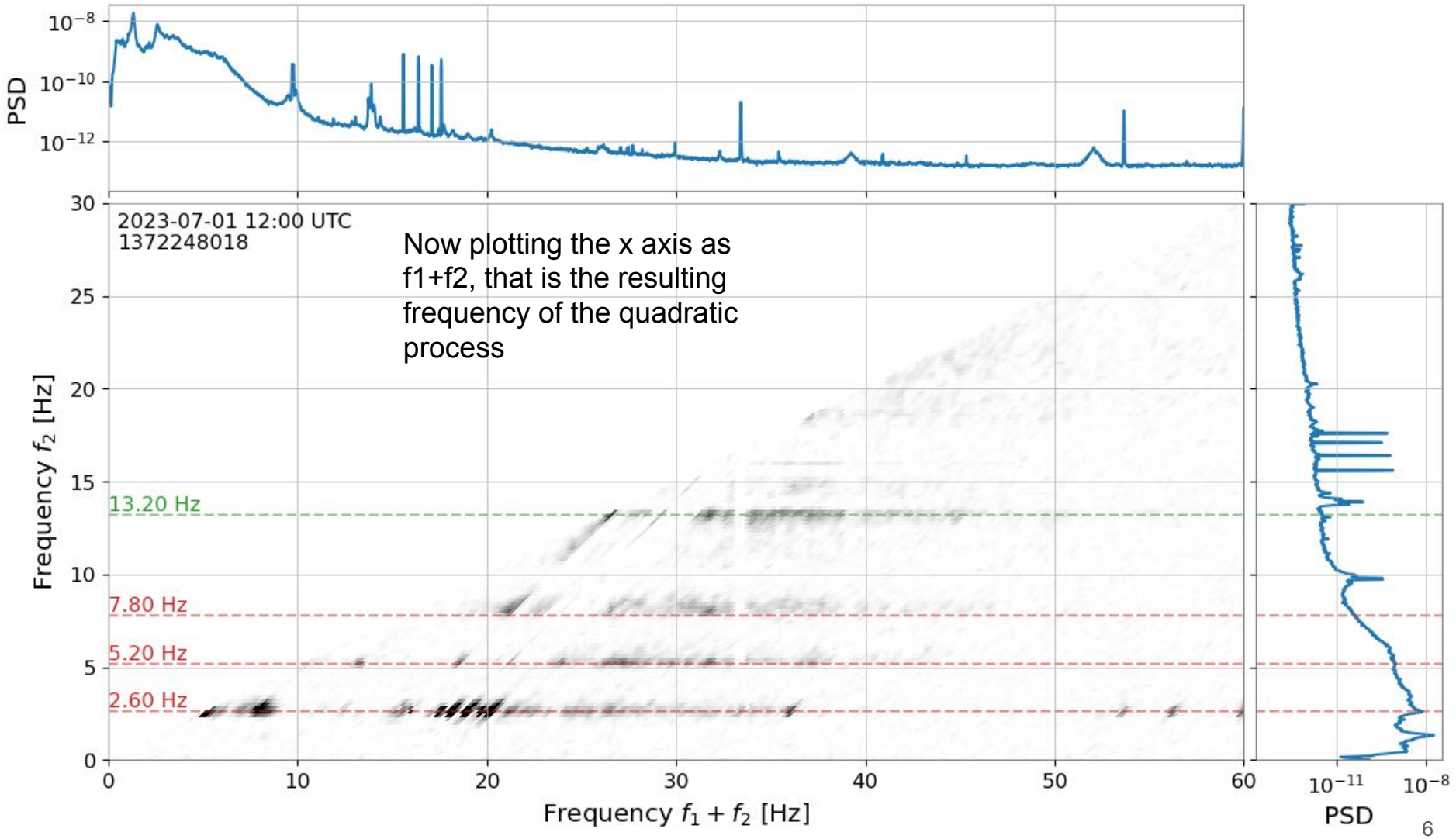
So in the following we plot only one area (inside the red triangle), all the others have equivalent information

DARM bicoherence

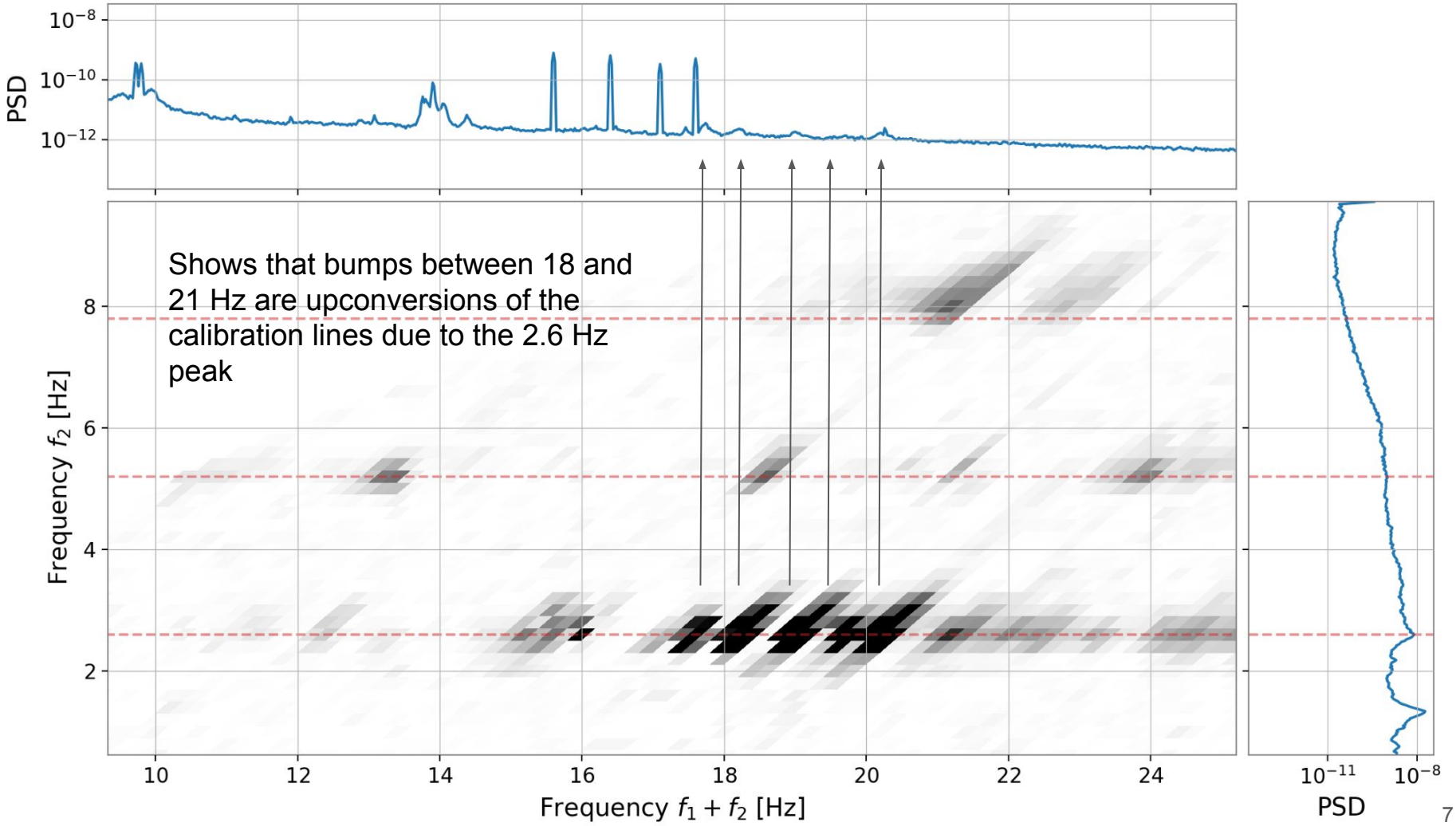
Note that if the bicoherence is high at a point (f1,f2), that means that the noise in DARM at f1+f2 is correlated with the “product” of noise at f1 and at f2



DARM bicoherence / f1+f2 horizontal axis



DARM bicoherence: calibration lines



How did I convince myself it's not an artifact /1

Repeated the bicoherence computation on multiple times, 3600 second long, found always the same pattern:

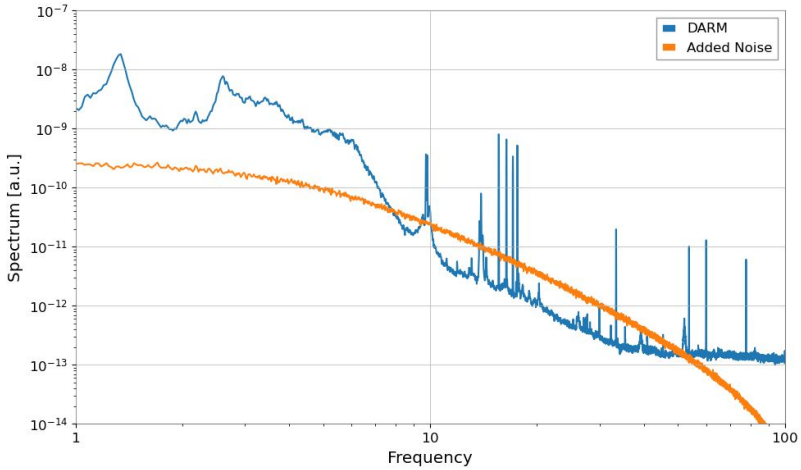
GPS: 1372248018, 1372064418, 1371945618, 1371798012

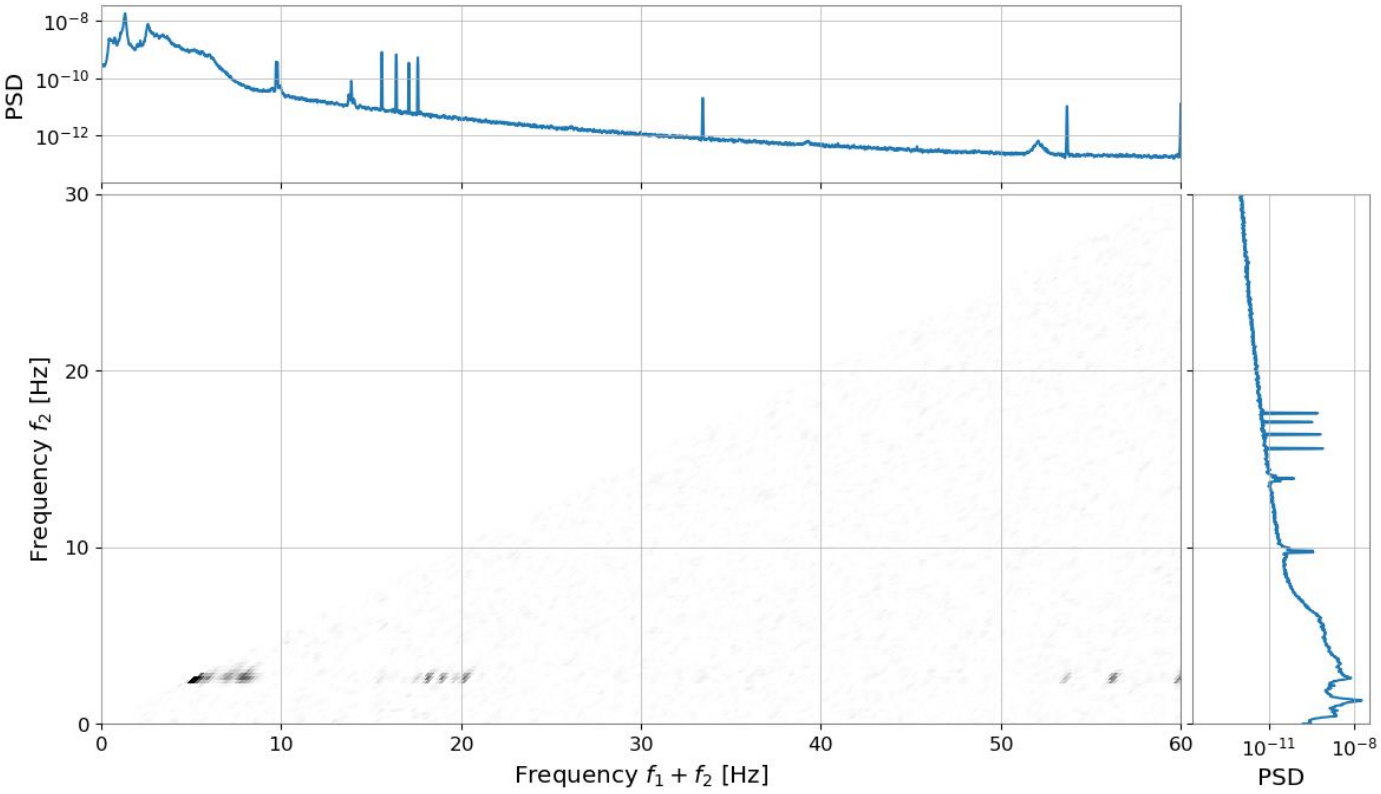
Very sensitive to glitches, so need to find times without large glitches

Plus numerical experiments to interpreter what we see in DARM, see next slides

Numerical experiments /1

Added some stationary noise to DARM: to see if the bicoherence is an artifact due to the presence of a peak at 2.6 Hz:
NO, IT'S NOT

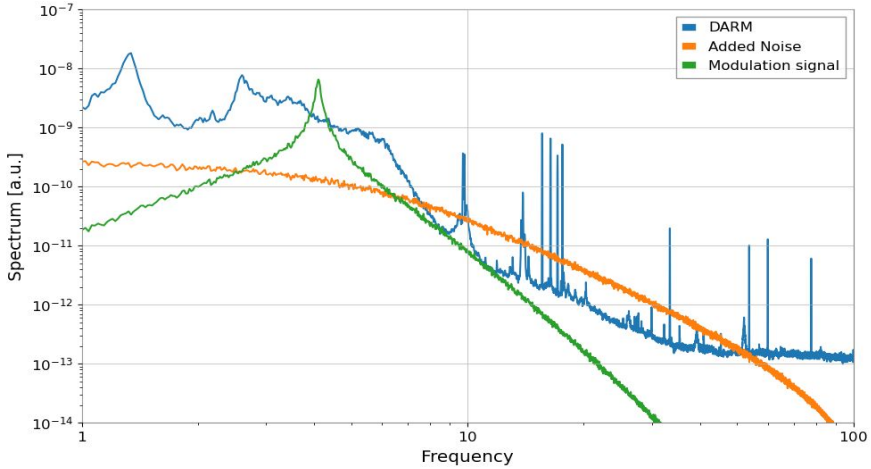


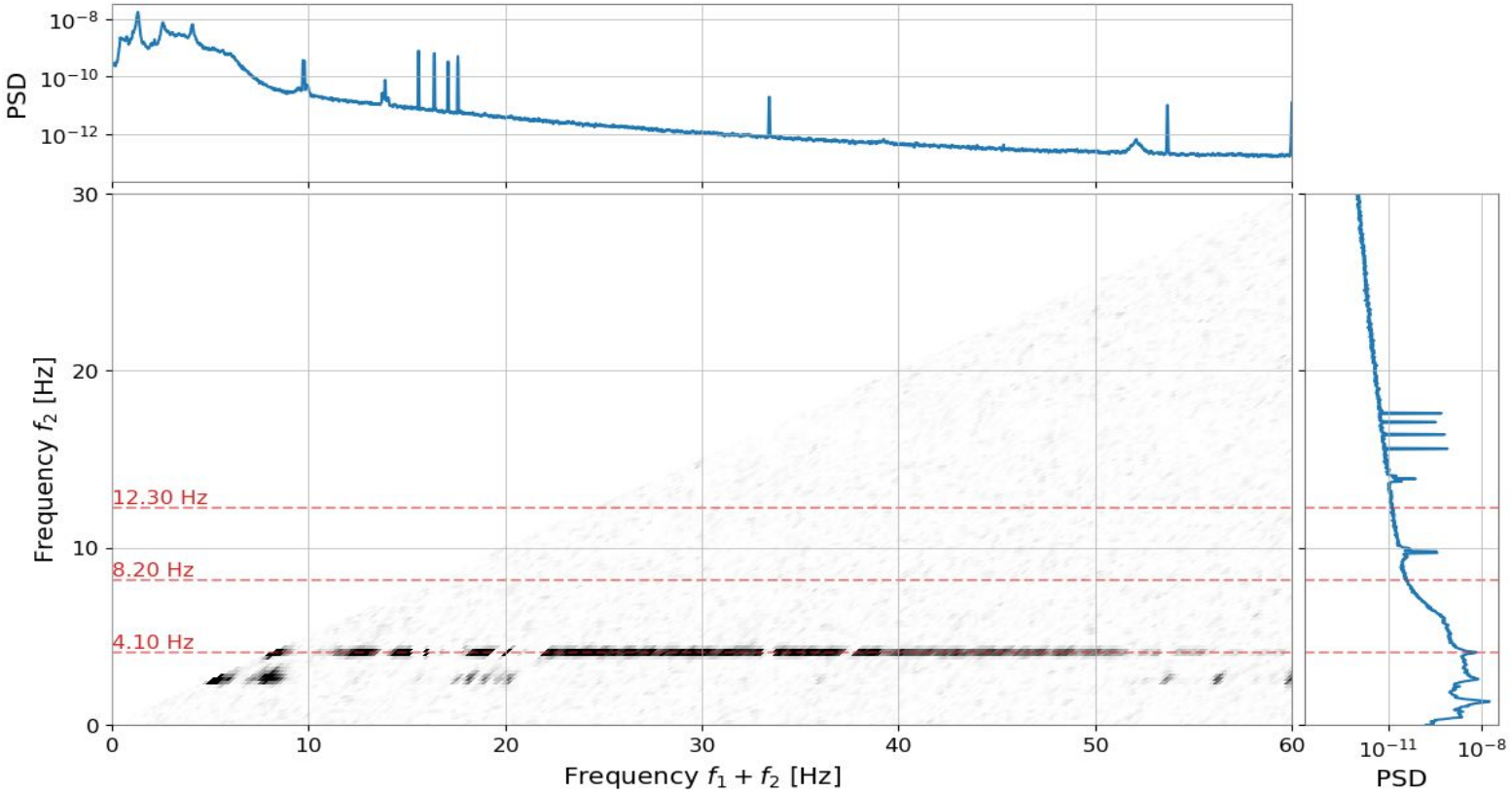


Added some stationary noise to DARM that covers the DARM noise, and the bicoherence disappears, no it's not due to some numerical issue when there is a large DARM peak at 2.6 Hz

Numerical experiments /2

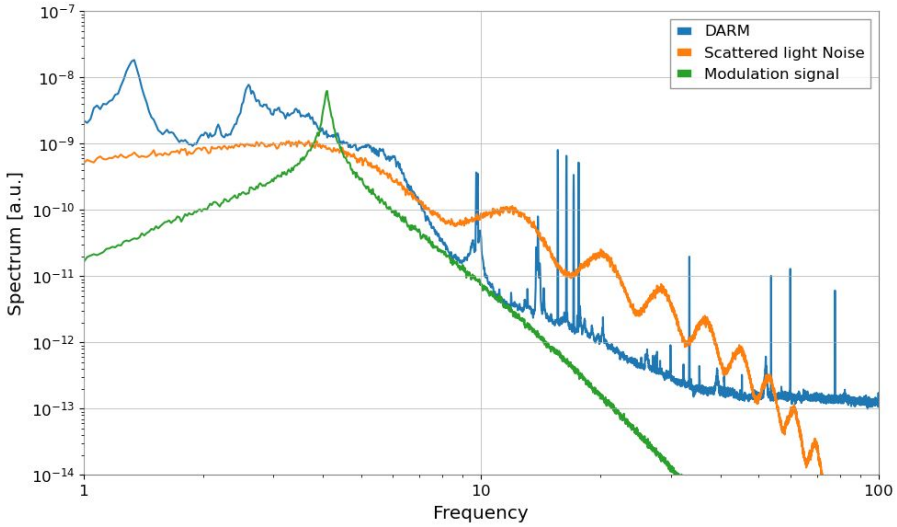
Added modulated noise to DARM: a peak at 4.1 Hz that modulates broad-band noise: bicoherence shows pattern similar to what we get with real DARM

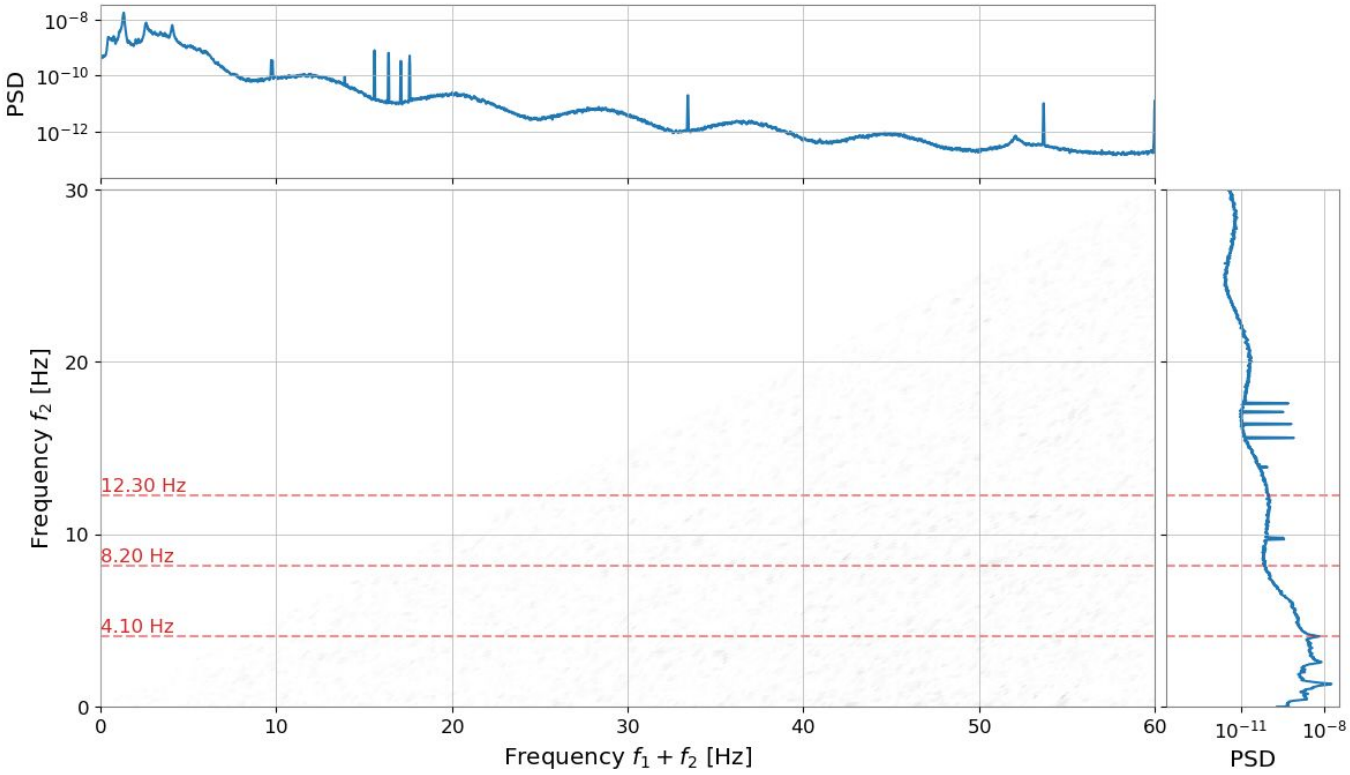




Numerical experiments /3

Added scattered light noise to DARM: a peak at 4.1 Hz deeply modulated with a scattered light sin() model: **no bicoherence**





This process seems to be too non linear to be picked up by the bicoherence

References

Choudhury, S.M., Shah, S.L., Thornhill, N.F. (2008). Bispectrum and Bicoherence. In: Diagnosis of Process Nonlinearities and Valve Stiction. Advances in Industrial Control. Springer, Berlin, Heidelberg. https://doi.org/10.1007/978-3-540-79224-6_3

Peter Zsolt Poloskei, Gergely Papp, Gabor Por, Laszlo Horvath, Gergo I. Pokol, Bicoherence analysis of nonstationary and nonlinear processes, https://arxiv.org/abs/1811.02973

HOSA - Higher Order Spectral Analysis Toolbox by Ananthram Swami
https://www.mathworks.com/matlabcentral/fileexchange/3013-hosa-higher-order-spectral-analysis-toolbox

YOUNG C.KIM AND EDWARD J.POWERS, IEEE TRANSACTIONS ON PLASMA SCIENCE, VOL. PS-7, NO. 2, JUNE 1979
Digital Bispectral Analysis and Its Applications to Nonlinear Wave Interactions
https://ieeexplore.ieee.org/document/4317207

Python code for the analysis adapted from Higher Order Spectrum Estimation toolkit
https://github.com/synergetics/spectrum